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ON AN EXTENSION OF HOLDITCH'S THEOREM.

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The demonstration of Elliott's extension of Holditch's Theorem in polar co-ordinates is interesting on account of the simple form of the differential expression for the *associate* area, and because the sectorial area swept over by the respective *radii vectores* are sometimes more easily investigated than the corresponding areas of rectangular co-ordinates. The theorem can be demonstrated in polar without the aid of the rectangular co-ordinates, but the reduction is more lengthy.

The following is perhaps the simplest method of reaching the result. In the expression (Williamson's Integral Calculus, p. 209)

$$\begin{aligned} (m+n)^2 y\,dx &= (m+n)(my_1dx_1 + ny_2dx_2) - mn(y_2-y_1)\,d(x_2-x_1), \\ (m+n)^2 x\,dy &= (m+n)(mx_1dy_1 + nx_2dy_2) - mn(x_2-x_1)\,d(y_2-y_1); \end{aligned}$$

write for brevity $m = (m+n)\,x'$ and $n = (m+n)\,x$,

and substitute

$$\begin{aligned} y &= \rho \sin\theta, & dy &= \sin\theta\,d\rho + \rho\cos\theta\,d\theta; \\ x &= \rho\cos\theta, & dx &= \cos\theta\,d\rho - \rho\sin\theta\,d\theta; \\ y_1 &= \rho_1\sin\theta_1, & dy_1 &= \sin\theta_1\,d\rho_1 + \rho_1\cos\theta_1\,d\theta_1; \\ x_1 &= \rho_1\cos\theta_1, & dx_1 &= \cos\theta_1\,d\rho_1 - \rho_1\sin\theta_1\,d\theta_1; \\ y_2 &= \rho_2\sin\theta_2, & dy_2 &= \sin\theta_2\,d\rho_2 + \rho_2\cos\theta_2\,d\theta_2; \\ x_2 &= \rho_2\cos\theta_2, & dx_2 &= \cos\theta_2\,d\rho_2 - \rho_2\sin\theta_2\,d\theta_2; \\ y_2 - y_1 &= \lambda\sin\omega, & d(y_2-y_1) &= \sin\omega\,d\lambda + \lambda\cos\omega\,d\omega; \\ x_2 - x_1 &= \lambda\cos\omega, & d(x_2-x_1) &= \cos\omega\,d\lambda - \lambda\sin\omega\,d\omega; \end{aligned}$$

where λ is the variable length of the moving line, and ω the angle which it makes with the initial line.

Subtracting one equation from the other, we have

$$\rho^2 d\theta = x\rho_2^2 d\theta_2 + x'\rho_1^2 d\theta_1 - xx'\lambda^2 d\omega;$$

or

$$dP = x\,dC' + x'\,dC - xx'\,dA,$$

using P, C, C', and A to represent the respective areas.

Now let λ make a finite shift; then

$$\int_{\theta'}^{\theta''} dP = x\int_{\theta_2'}^{\theta_2''} dC' + x'\int_{\theta_1'}^{\theta_2''} dC - xx'\int_{\omega'}^{\omega''} dA$$

is the general equation connecting the sectorial areas swept over by ρ, ρ_1, ρ_2, and the associate area A, the latter being determined by the curve $\lambda = f(\omega)$.

If we integrate for a complete circuit, we obtain

$$P = xC' + x'C - xx'A,$$

the different cases of which are noticed in Williamson's Calculus.

The mean value of P for $0 < x < 1$, since $x + x' = 1$, is

$$M = C'\int_0^1 x\,dx + C\int_0^1 x'\,dx' - A\int_0^1 x(1 - x)\,dx$$

$$= \tfrac{1}{2}(C + C') - \tfrac{1}{6}A,$$

the familiar form of the true prismoid formula for the mean area, where C and C' are the end areas, and A the area of the base of the associate cone; the equation to its boundary being $\lambda = f(\omega)$, where λ is the projection on the base of the curved surface generator, and ω the angle through which it turns. If, however, the extremities and the line oscillate back to their former positions without revolving, then

$$C = 0, \quad C' = 0, \quad \text{and} \quad A = 0,$$

and also $P = 0$, because the positive and negative areas destroy each other in complete circuit, and the point P either retraces its path or crosses it before returning to its starting point, thus dividing the closed area into equal positive and negative portions.

If then, instead of integrating for a complete circuit of P, we integrate only for a complete circuit of any one loop, we obtain the area of that loop (and thus of the whole closed curve, provided none of the loops overlap or have a portion of their area in common), the data being such that we can obtain the limits of θ_1, θ_2, and ω, which correspond to the shift of λ as P describes the loop, the values of $\int dC$, $\int dC'$, and $\int dA$ being obtainable.